Statistics of PMD in Recirculating Loops

Mats Petersson, Claudio Vinegoni, Henrik Sunnerud, and Magnus Karlsson

***Abstract*—The statistical distribution of polarization-mode dispersion in a recirculating loop is investigated. Numerical simulations are performed in both Jones and Stokes space and are verified by experiments. The probability distribution of the differential group delay (DGD) is obtained numerically, theoretically, and experimentally. As the number of circulations increase, the probability density function of the DGD approaches a uniform distribution.**

***Index Terms*—Differential group delay (DGD), polarization-mode dispersion (PMD), probability density function (pdf), recirculating loop.**

I. INTRODUCTION

SINCE THE polarization-mode dispersion (PMD) in an optical fiber drifts randomly with time, wavelength, and fiber length, it is treated as a stochastic phenomenon. The differential group delay (DGD), which is the time delay between the two orthogonal principal states of polarization (SOP), can be regarded as a random variable with a Maxwellian probability density function (pdf), and with an expectation value that increases with the square root of the fiber length [1].

Recirculating loops are frequently used in research laboratories for emulating long-haul straight line transmission links [2]. However, since the recirculating loop is a periodic structure, it does not correctly resemble the statistical properties of a randomly birefringent straight-line link. As a result, the average DGD grows approximately linearly with the fiber length. It has been shown that it is possible to reproduce the accurate Maxwellian DGD statistics by scrambling the polarization between each circulation inside the loop [3]. However, since conventional recirculating loops still are used, it is interesting to investigate their PMD statistics in order to understand the difference between the two methods. In this letter, we investigate the probability distribution of the DGD in such recirculating loops. A polarization controller (PC) inside the loop is scrambled between each measurement of the accumulated DGD, but remains unchanged as the light circulates inside the loop. The distribution of the accumulated DGD after N circulations is derived by simulations, theory, and experiments.

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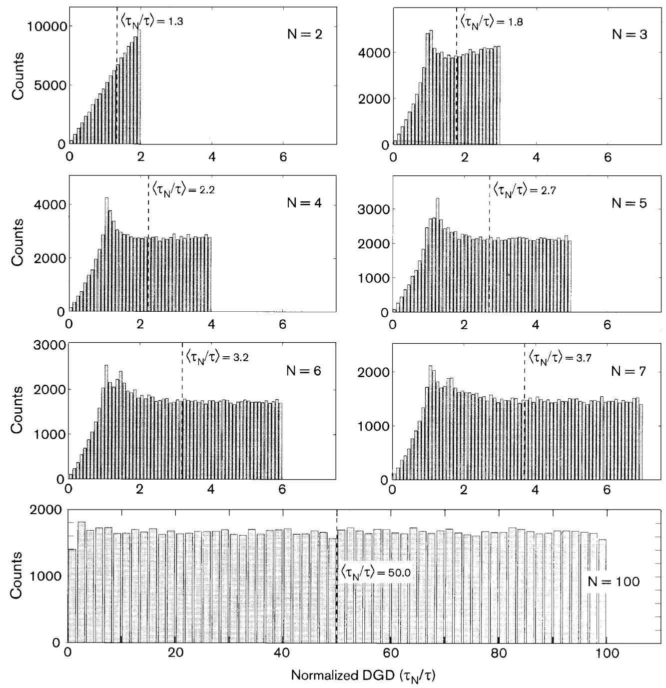

Fig. 1. Histograms of the accumulated DGD for two to seven, and 100 circulations through a recirculating loop. Number of runs 10^5. The dashed line indicates the expectation value $\langle \tau_N \rangle$.

II. MATHEMATICAL APPROACH—JONES SPACE

The transformation of an optical field during transmission in a fiber can be described by the Jones matrix T according to $\mathbf{E}_{\rm out} = T\mathbf{E}_{\rm in}$, where $\mathbf{E}_{\rm in}$ and $\mathbf{E}_{\rm out}$ are the input and output optical field components, respectively. In Jones space, a birefringent element and a PC can be described mathematically by the Jones matrices $T_{\rm PMD}$ and $T_{\rm PC}$ (polarization-dependent loss (PDL) not considered)

$$T_{\rm PMD}(\omega) = \begin{pmatrix} e^{i\frac{\tau\omega}{2}} & 0 \\ 0 & e^{-i\frac{\tau\omega}{2}} \end{pmatrix} \quad (1)$$

$$T_{\rm PC}(\theta,\phi) = \begin{pmatrix} \cos\theta e^{i\phi} & -\sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta e^{-i\phi} \end{pmatrix} \quad (2)$$

where τ is the DGD of the birefringent element. Equations (1) and (2) imply that the PMD is fixed, while the PC can be changed arbitrarily by changing θ and ϕ. For light passing through the birefringent element and the PC, the resulting Jones matrix is $T_{\rm loop} = T_{\rm PMD}T_{\rm PC}$ and the total Jones matrix after N circulations through the loop is $T_{\rm loop}^N$. The accumulated DGD τ_N of the loop is then given by the difference of the imaginary parts of the eigenvalues of the matrix product $T_{\rm loop}^{N'}(T_{\rm loop}^N)^{-1}$ [4]. To accomplish correct polarization scrambling with the PC, the angles θ and ϕ should have the pdfs $p(\theta) = \sin(2\theta)$; $\theta \in [0, \pi/2]$ and $p(\phi) = 1/(2\pi)$; $\phi \in [0, 2\pi]$ [5]. Fig. 1 shows the computed histograms of the accumulated DGD from two up to seven, and 100 circulations through the loop for 10^5 runs. For $N = 2$ the probability distribution of τ_N is a linearly increasing function, which is identical to the theory for two concatenated PMD sections with equal amount and random directions of the PMD [6], [7]. For $N \geq 3$, the distribution of τ_N is approximately quadratic for $0 < \tau_N < \tau$ and almost uniform for $\tau < \tau_N < N\tau$. As the number of circulations through the recirculating loop increases, the DGD seems to approach a uniform distribution with the expectation value $\langle \tau_N \rangle$ increasing linearly with N. This will be further investigated in Section III.

III. Mathematical Approach—Stokes Space

It is convenient to consider polarization issues in Stokes space, which gives a geometric interpretation of the SOP. In Stokes space, each point on the unit sphere, the Poincaré sphere, represents a specific SOP. PMD is characterized by the PMD vector $\boldsymbol{\tau}$, and the DGD equals the length of the PMD vector $\tau = |\boldsymbol{\tau}|$. In the loop, $\boldsymbol{\tau}$ is fixed, and its length and direction are given by the birefringent element. The effect of a PC corresponds to a rotation of the SOP an angle α around a rotation unit vector $\mathbf{r}$. The amount and direction of the rotation are determined by the PC setting. One can show that in order to accomplish correct polarization scrambling, the angle β between $\mathbf{r}$ and $\boldsymbol{\tau}$ should be distributed as $p(\beta) = \sin(\beta)/2$; $\beta \in [0, \pi]$, and the rotation angle α be distributed as $p(\alpha) = (2/\pi)\sin^2(\alpha/2)$; $\alpha \in [0, \pi]$ [5] when $\mathbf{r}$ is allowed in any direction in Stokes space. The effects of the birefringent element and the PC can be described mathematically by a Müller matrix M according to $\mathbf{s}_{\text{out}} = M\mathbf{s}_{\text{in}}$, where $\mathbf{s}_{\text{in}}$ and $\mathbf{s}_{\text{out}}$ are the input and output SOP, respectively. The Müller matrices for the PMD and the PC can be expressed as $M_{\text{PMD}}(\omega) = \exp(\omega\boldsymbol{\tau}\times)$ and $M_{\text{PC}}(\alpha, \mathbf{r}) = \exp(\alpha\mathbf{r}\times)$ [8], where $\boldsymbol{\tau}\times$ and $\mathbf{r}\times$ are the cross-product matrices of $\boldsymbol{\tau}$ and $\mathbf{r}$, respectively, according to

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \Leftrightarrow v\times = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}. \quad (3)$$

Analogous to Jones space, the total Müller matrix after N circulations through the loop is M_{loop}^N, where $M_{\text{loop}} = M_{\text{PMD}} M_{\text{PC}}$. The accumulated PMD vector $\boldsymbol{\tau}_N$ after N circulations through the loop is obtained from $\boldsymbol{\tau}_N\times = M_{\text{loop}}^{N'}(M_{\text{loop}}^N)^{-1}$, and τ_N can be computed.

After N passes through the loop, $\boldsymbol{\tau}_N$ is the sum of N PMD vectors. Since polarization scrambling is not performed during transmission, the nth PMD vector is rotated an angle α around the vector $\mathbf{r}$ compared to the $(n-1)$th PMD vector. Thus, the accumulated PMD vector will not walk randomly in three dimensions as in a real transmission fiber [6], but will evolve spirally through Stokes space, according to the concatenation rule [6], [8]

$$\boldsymbol{\tau}_N = \sum_{k=0}^{N-1} e^{k\alpha\mathbf{r}\times}\tau \quad (4)$$

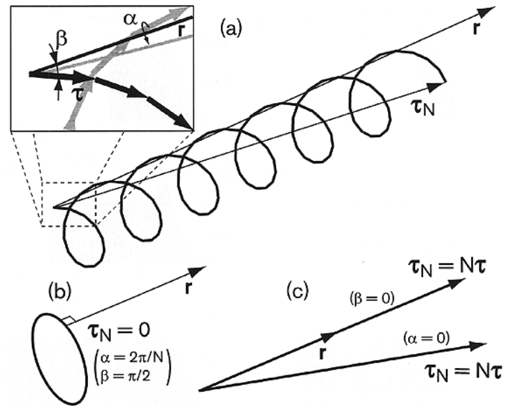

Fig. 2. (a) Typical evolution of the accumulated PMD vector $\boldsymbol{\tau}_N$ in Stokes space ($N = 100$, $\beta = \pi/3$, $\alpha = \pi/8$). Here, $|\boldsymbol{\tau}_N| \approx N|\boldsymbol{\tau}|/2$. (b) The evolution resulting in $|\boldsymbol{\tau}_N| = 0$ ($N = 100$, $\beta = \pi/2$, $\alpha = \pi/50$). (c) Two examples resulting in $|\boldsymbol{\tau}_N| = N\tau$ ($N = 100$, $\beta = 0$ and $\alpha = 0$). The length of $\mathbf{r}$ is modified for illustrative reasons.

as illustrated in Fig. 2 for $N = 100$. Depending on the angle β, the spiral will be more or less extended. When $\beta = \pi/2$, $\boldsymbol{\tau}_N$ describes a circular movement perpendicular to $\mathbf{r}$. If $N\alpha$ is modulo 2π at the same time, the accumulated DGD becomes zero, as shown in Fig. 2(b). On the other hand, $\boldsymbol{\tau}_N$ will evolve along a straight line if $\beta = 0$ or π and, alternatively, if $\alpha = 0$. This results in the maximum possible accumulated DGD $\tau_N = N\tau$, shown in Fig. 2(c). By decomposing $\boldsymbol{\tau}$ in components parallel and perpendicular to $\mathbf{r}$, one can show that τ_N can be expressed as

$$\tau_N^2 = \tau^2 \left[N^2\cos^2(\beta) + \sin^2(\beta)\frac{\sin^2\left(\frac{N\alpha}{2}\right)}{\sin^2\left(\frac{\alpha}{2}\right)} \right]. \quad (5)$$

By using (5) and the distributions of α and β, the probability distributions of τ_N from two up to seven, and 100 circulations through the loop were obtained and the results were identical to the results obtained from simulations in Jones space. It is also possible to derive the expectation value of the square DGD $\langle \tau_N^2 \rangle$, which becomes $\langle \tau_N^2 \rangle = \tau^2(N^2+2)/3$. Considering $N \to \infty$, the second term in (5) becomes negligible with respect to the first term, giving $\tau_N = N\tau|\cos(\beta)|$, which means τ_N is uniformly distributed between zero and $N\tau$, as seen for $N = 100$ in Fig. 1, since $|\cos(\beta)|$ is uniformly distributed between zero and one. As a result, the expectation value of the DGD $\langle \tau_N \rangle$ increases linearly as $\langle \tau_N \rangle = N\tau/2$ for large numbers of circulations through the loop.

IV. Experiments

The experimental setup is shown in Fig. 3. The loop was controlled by three acoustooptical switches (AOS). AOS1 filled the loop, AOS2 emptied the loop, and AOS3 controlled the output of the loop. A tunable laser, switched between two wavelengths separated 0.4 at 1550 nm, was used as the light source. The loop itself consisted of an electrically controlled PC, a polarization-maintaining fiber with a DGD of 1.6 ps, an erbium-doped fiber amplifier, a 1.3-nm optical filter (full-width at half-maximum), the AOS2, and a 12-km standard single-mode fiber. After transmission through the loop, the light was analyzed by a

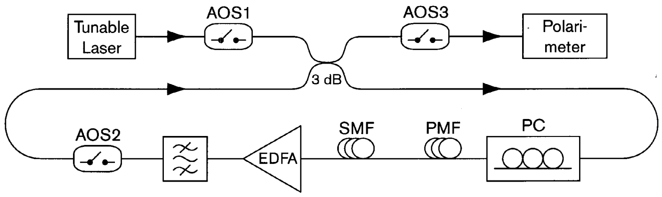

Fig. 3. Recirculating loop used in the experiments.

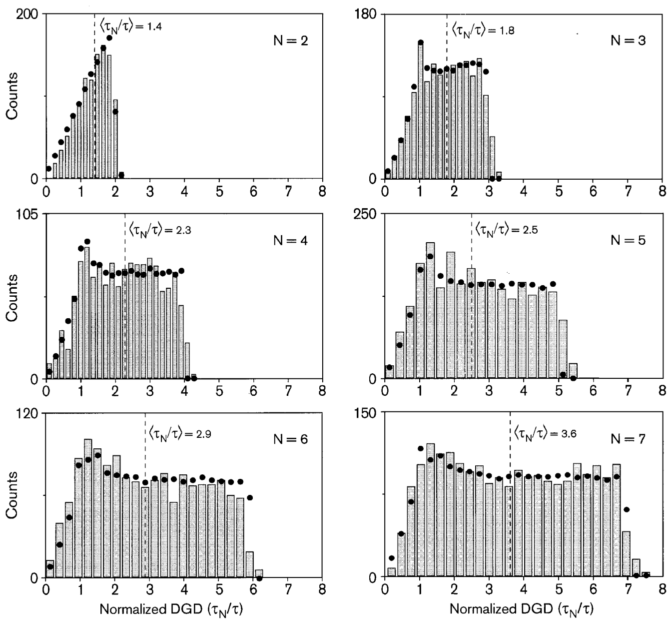

Fig. 4. Measured histograms of the accumulated DGD for two to seven circulations through the experimental recirculating loop. Number of measurements around 2000. The dots indicate the corresponding theoretical values (numerically obtained) of the bin widths used in the experiments. Dashed line indicates the expectation value $\langle \tau_N \rangle$.

polarimeter. The measured average PDL of the loop was 0.2 dB and the fluctuation of τ was 0.1 ps.

Measurements from two up to seven circulations in the loop were performed with the PC uniformly scrambled between the measurements. The DGD was computed according to the Jones matrix eigenanalysis and roughly 2000 samples of the DGD were measured for each number of circulations through the loop. The time to obtain a single DGD sample was about 1 min, and the obtained measurement data are the result of several weeks of continuous measurements. The results are shown in Fig. 4 together with the corresponding theoretical values (numerically obtained) of the bin widths used in the experiments. The agreement between the experimental and the theoretical results are excellent. The reason for measuring $\tau_N > N\tau$ in the experiments can be explained by the fluctuations of the measured τ. However, it is clear that the experiments verify the theory.

V. Conclusion

The statistical distribution of the PMD in a recirculating loop has been theoretically and experimentally investigated and an analytical expression of the accumulated DGD has been derived. It was found that the accumulated PMD vector evolves spirally through Stokes space. As a result, the probability distribution of the accumulated DGD becomes a linearly increasing function for two passes through the loop, but for increasing number of circulations, the DGD approaches a uniform distribution. The expectation value of the DGD increases linearly with the number of circulations through the loop, in contrast to a straight line transmission link, where the expectation value of the DGD grows as the square root of the link length.

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