The Statistics of Polarization-Dependent Loss in a Recirculating Loop

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***Abstract*—In this paper, we study the statistical distribution of the accumulated polarization-dependent loss (PDL) in a recirculating loop. The distribution is studied both via numerical simulations and with analytical theory and very good agreement is found between the two. In addition, we have experimentally obtained the probability density distribution for the PDL and, even in this case, we find good agreement with the predicted one. The mean accumulated PDL is found to grow linearly with the number of circulations N in contraposition to a straight-line system. Moreover, the statistical distribution tends to become uniformly distributed as N is increasing. Finally, the statistics of the PDL for a recirculating loop, when considering small values of PDL, is found to be equal to the statistics of the differential group delay for recirculating loops.**

***Index Terms*—Optical fibers, optical fibers polarization, polarization-dependent loss (PDL), polarization-mode dispersion (PMD), recirculating loop.**

I. Introduction

IT is well known that single-mode optical fibers (SMF) support two distinct orthogonal polarization modes. Normally, the attenuations of these two modes are identical. This means that the total attenuation of the fiber can be considered insensitive to the polarization state at the fiber input [1]. But optical links are made up of more than simple fibers and usually different components are required, i.e., couplers, isolators, filters, multiplexers, and amplifiers. All these components present indeed a small anisotropy and are subject to polarization-dependent losses (PDL) that eventually will affect the system's performances. This is analogous to the situation present for the polarization-mode dispersion (PMD) that is well known to introduce severe system penalties at high bit rates. In this respect, the importance of evaluating the global PDL of a system link, arises straightforwardly.

A first notable result that immediately occurs when considering polarization-dependent losses, is that the total PDL of a series of concatenated elements is usually different from the sum of each single PDL-element contribution. The reason has to be ascribed to the fact that the polarization sensitive axes of the single components are not always necessarily aligned with each other; therefore, the resulting total PDL depends on the relative orientations of the PDL axis at each connection [2].

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But what are the values of the PDL for typical optical components? Usually, components like isolators, couplers, and Erbium doped fiber amplifiers (EDFAs), can have a PDL up to 0.3 dB, but this values are strongly dependent on the environmental conditions (i.e., stress and/or temperature), with fluctuations of the order of 0.1 dB. This uncertainty and the fact that we can have, in general, a randomized alignment between the relative axis orientations of the single PDL elements, suggest the importance in finding a statistical description of the total accumulated PDL, in order to have a prediction of the global attenuation statistics.

Studies in this direction, after the first seminal work of Gisin [1], were recently published and the total accumulated PDL for an optical communication system is found to be Maxwellian distributed (when expressed in decibels) with the accumulation of the mean PDL growing linearly with the system length [3], [4].

Things get even more interesting and complicated when PMD is present in the system, a situation that is quite often the norm in a real link. In this case, the total concatenated PDL fluctuates in time or with changes in wavelength [2]. This implies that the resulting power fluctuations along the link, can introduce a nonnegligible deterioration in the optical signal-to-noise ratio (OSNR), affecting the total accumulated PDL distribution [5].

But even the PMD distribution is in fact altered by the presence of the PDL, for the reason that the interplay between the PDL and the PMD is quite subtle as evidenced by Gisin *et al.* [6], making their contribution not separable [7]–[9]. In essence what is happening is that the combined presence of PMD and PDL may introduce fluctuations larger than those estimated by root mean square rules. This was recently confirmed both by simulations [10] and experimentally [6], [11].

For all the fore-mentioned reasons it is clear in final instance that nowadays there is a strong actual need to study the distributions of the PMD and the PDL, both combined and separately. Unfortunately, when we come to the issue of studying statistical properties of straight-line optical links, we immediately face the problem that it is not easy to access and/or reproduce sufficiently long links. Moreover it is not realistic to imagine to have controls on each single links' component parameters, like PDL, EDFA, PMD, etc. A natural way, and less expensive, to reproduce on the laboratory table an optical fiber communication link, consist in considering a recirculating loop [12]. This was done quite frequently in the past and even today it is a quite standard way to emulate long links. But, due to the periodicity of the system, polarization phenomena like for example PMD (and PDL, as it will be demonstrated in the paper) are quite different from the one present in an optical fiber link. In fact the distribution of the PMD can be found unrealistically distributed in loops in which no inter-loop polarization decorrelation exist [13], [14]. This problem can give rise to misleading phenomena that are not present in real straight-line systems, but are instead intrinsically related to the loop periodicity.

We investigated this behavior in detail in a recent work [14], focusing on the probability distribution function of the differential group delay (DGD) in a loop. We obtained both numerically, theoretically, and experimentally the probability distribution of the DGD, and we found that as the number of circulations increase, the probability density function (pdf) of the DGD approaches a uniform distribution.

Concerning the PDL, an analysis of the evolution of the signal's state of polarization (SOP) was recently done [15], [16] for the case of a loop, in which polarization-dependent losses are present. It was found indeed that even the PDL plays a major role in the performances of the loop, and in determining the evolution of the signal's SOP. In particular the polarization state evolves in a spiral way on the Poincaré sphere with different types of spirals, each one associated with different systems performances [15].

Regarding the more general problem of determining the statistical distribution of the PDL in a recirculating loop, no systematic study was ever undertaken up to now, and this is the actual object of study of the present work.

Hereby we present both a theoretical, a numerical, and an experimental study of the statistical distribution of the accumulated PDL in a recirculating loop. After this introductory preamble, in Section II, we introduce first the concept of PDL, and then we determine via numerical simulations the PDL statistical distribution. In addition we demonstrate a theoretical model capable of an analytical explanation of the PDL distribution, showing that this distribution coincides with the numerically calculated one. In Section III, we describe the setup used to make the measurements and we present and discuss the obtained results. The good agreement between the experimental data and the theoretical, confirms the validity of our model and of the numerical simulations.

II. Theory

Let us review briefly the definition and properties of polarization-dependent loss. The PDL of an optical component is defined as the ratio of the maximum over the minimum optical transmission coefficient ($\mathrm{T_{max}/T_{min}}$), of the component. In terms of the (unit length, dimensionless) Stokes vector of the light $\boldsymbol{s}$, the transmission coefficient for a PDL element can be expressed as

$$T = p + \boldsymbol{s} \cdot \boldsymbol{p} \tag{1}$$

where the scalar p denotes the polarization independent power transmission, and the vector $\boldsymbol{p}$ denotes the polarization-dependent power transmission (note that this relation essentially reflects the first row of the Mueller matrix of the PDL element [17]). Thus the max/min transmissions are given by $T_{max/min} = p \pm |\boldsymbol{p}|$, and the PDL (in linear units) is given by

$$\mathrm{PDL} = \frac{p + |\boldsymbol{p}|}{p - |\boldsymbol{p}|}. \tag{2}$$

The vector $\boldsymbol{p}$ is directed in the direction of maximum transmission (minimum PDL) [18]. Expressed in decibel units the PDL is equal to

$$\mathrm{PDL_{dB}} = \gamma = 10 \log \left(\frac{T_{\max}}{T_{\min}} \right). \tag{3}$$

One may define a PDL vector as the ratio between the polarization-dependent and independent losses, i.e., $\boldsymbol{\Gamma} = \boldsymbol{p}/p$ as introduced in [1]. However, we believe it is valuable (both from a physical and as we will see also mathematical point of view) to keep both the polarization-dependent and independent parts separated, and consider the properties of them both.

A. The Concatenation Rule

We will first consider the concatenation rule for two PDL elements following each other. The transmission is assumed to be given by (1) with $p_{1,2}$ and $\boldsymbol{p}_{1,2}$ in the respective elements. The corresponding Jones matrices can be written as

$$M_1 = a_1 I + \boldsymbol{a}_1 \cdot \boldsymbol{\sigma} \tag{4}$$

and similar for M_2 by replacing the indices. Here $\boldsymbol{\sigma}$ is the Pauli spin vector [1], I is the unit matrix, and the vector $\boldsymbol{a}$ is a unit vector. The relation to the transmission coefficients p and $\boldsymbol{p}$ defined above, can be written as $p_1 = a_1^2 + \boldsymbol{a}_1 \cdot \boldsymbol{a}_1$ and $\boldsymbol{p}_1 = 2a_1\boldsymbol{a}_1$. With these definitions we are ready to compute the concatenation rule. Assuming the total transmission Jones matrix is $M_{12} = M_2 M_1$ so that the light hits element 1 first, then the corresponding transmission is $T_{12} = p_{12} + \boldsymbol{s} \cdot \boldsymbol{p}_{12}$, where the polarization independent transmission is

$$p_{12} = p_1 p_2 + \boldsymbol{p}_1 \cdot \boldsymbol{p}_2 \tag{5}$$

and the polarization-dependent transmission vector is given by

$$\boldsymbol{p}_{12} = p_1 \boldsymbol{p}_2 + p_2 \boldsymbol{p}_1 + 2\boldsymbol{a}_1 \times (\boldsymbol{a}_1 \times \boldsymbol{p}_2). \tag{6}$$

We believe this is the first time the PDL concatenation rule is given in the separated form, which can be quite valuable. The concatenation rule given by Gisin [1] is found from this result by defining $\boldsymbol{\Gamma}_k = \boldsymbol{p}_k/p_k$ for $k = (1, 2, 12)$ and using the above (5), (6).

In most cases of practical interest in fiber communications, the PDL is rather low, and this concatenation rule simplifies considerably. For example, if the PDL of the two concatenated elements are equal to each other and less than 3 dB, then the ratio between the polarization-dependent and the polarization independent transmission of the elements $|\boldsymbol{\Gamma}_{1,2}| = |\boldsymbol{p}_{1,2}|/p_{1,2}$ is less than 0.33, and the two last terms in (5), (6) contribute with less than 10% and can be neglected. In such a situation the concatenation rule simplifies to

$$p_{12} = p_1 p_2 \tag{7}$$

$$\boldsymbol{p}_{12} = p_1 \boldsymbol{p}_2 + p_2 \boldsymbol{p}_1 \tag{8}$$

or, in terms of the PDL vector, $\boldsymbol{\Gamma}_{12} = \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_2$. This is obviously very straightforward to generalize to a case with many small PDL elements, which then will be the vector sum of the individual PDL vectors. It follows that the polarization independent losses accumulate as a product, just as conventional scalar transmission coefficients. The PDL vector concatenation rule is a very important result also for the statistical properties of the PDL vector. In fact, since a similar concatenation rule holds for the PMD vector, we can directly map many of the results for the PMD vector to the PDL vector case. For example, for a large number of randomly oriented PDL-vectors, the length of the PDL-vector will be Maxwellian distributed and its average will scale with the square root of the number of elements. Moreover, since the $\mathrm{PDL}_{\mathrm{dB}} = \gamma$ is approximately proportional to the length of the PDL vector $|\mathbf{\Gamma}|$ up to around 6 dB, ($\gamma \approx 20|\mathbf{\Gamma}|/\ln(10) = 8.68|\mathbf{\Gamma}|$), then γ will obey the same statistics as the differential group delay (DGD) in a random PMD fiber. This has been noted previously by a number of authors, e.g., [3], [4].

B. The Circulating Loop Case

The case of the PMD statistics in a circulating loop was recently investigated [14] both theoretically and experimentally, but we reproduce it here in the PDL-case for convenience. In a circulating loop, the transfer matrix for the PDL will be repeated each lap, after having undergone a random polarization change, modeled by a random Jones matrix. We call this Jones matrix M_{PC}, since in the laboratory it corresponds to a polarization controller. In Stokes space this polarization change will correspond to a random rotation of angle α, around a rotation unit vector $\boldsymbol{b}$. The above concatenation rule can now be applied to obtain the total PDL vector after N laps. To simplify this discussion, we project the PDL vector of one lap, $\mathbf{\Gamma}_{\mathrm{loop}}$ along and perpendicular to the rotation vector $\boldsymbol{b}$, which then gives

$$\Gamma_{\|} = \cos(\beta)\Gamma_{\mathrm{loop}} \tag{9}$$

$$\Gamma_{\perp} = \sin(\beta)\Gamma_{\mathrm{loop}} \tag{10}$$

where β is the angle between the PDL vector of one lap, and the rotation vector $\boldsymbol{b}$. After N laps, the components of the accumulated PDL vector $\mathbf{\Gamma}_N$ will be

$$\Gamma_{N,\|} = N\cos(\beta)\Gamma_{\mathrm{loop}} \tag{11}$$

$$\Gamma_{N,\perp} = \frac{\sin\left(N\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}\sin(\beta)\Gamma_{\mathrm{loop}}. \tag{12}$$

The case of $\mathrm{N} = 2$, is a bit special; the length $|\mathbf{\Gamma}_2|$ of two randomly oriented vectors will be distributed as a linear function [19]. For N large, the parallel component will dominate, so that $\mathbf{\Gamma}_N \cdot \approx \boldsymbol{b}\Gamma_{N,\|}$ and the pdf of Γ_N will be approximately equal to the PDF of $N\cos(\beta)\Gamma_{\mathrm{loop}}$, which is determined by the PDF of $\cos\beta$. It is well-known that the PDF of the projection of a constant-length, randomly directed vector on a given vector (such as the $\hat{x}$-component of a random vector, or as in our case, the $\boldsymbol{b}$-component of the $\mathbf{\Gamma}_{\mathrm{loop}}$-vector) is uniformly distributed (here "randomly directed" means uniformly distributed over all directions in 3-space). Hence the PDF of Γ_N will, for large values of N, be uniformly distributed. The average of the squared PDL vector after N laps is also straightforwardly computed; by averaging (11), (12). This gives us

$$\left\langle \Gamma_{N,\|}^2 \right\rangle = N^2 \left\langle cos^2(\beta) \right\rangle \Gamma_{\mathrm{loop}}^2 = \frac{N^2}{3}\Gamma_{\mathrm{loop}}^2 \tag{13}$$

$$\left\langle \Gamma_{N,\perp}^2 \right\rangle = \left\langle \left(\frac{\sin\left(N\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}\right)^2 \right\rangle \left\langle \sin^2(\beta)\right\rangle \Gamma_{\mathrm{loop}}^2 = \frac{2}{3}\Gamma_{\mathrm{loop}}^2. \tag{14}$$

To compute the average of the perpendicular component, we use the fact that the rotation angle α has the PDF $p(\alpha) = 2/\pi \sin^2(\alpha/2)$; $\alpha \in [0,\pi]$ [20]. Thus the average length squared of the PDL vector after N laps is given by

$$\left\langle \Gamma_N^2 \right\rangle = \frac{N^2+2}{3}\Gamma_{\mathrm{loop}}^2. \tag{15}$$

Since the PDL γ is linearly proportional to the length of the PDL vector in the regime we consider, the above conclusion for the modulus of the PDL vector Γ_N will also hold for the PDL γ.

C. Numerical Simulations

Our recirculating loop can be modeled as a polarization rotation along a selected arbitrary axis, followed by a PDL element (we assume in our model that no PMD is present in the loop). In a Jones matrix representation, the polarization rotation can be described by a Jones matrix M_{PC}, which in a general form can be written as

$$M_{\mathrm{PC}}(\theta,\phi) = \begin{pmatrix} \cos\theta e^{i\phi} & -\sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta e^{-i\phi} \end{pmatrix}.$$

Changing θ and ϕ, we can emulate the behavior of any loop transfer function.

For what concerns the PDL element, it can be simply represented by a Jones matrix M_{PDL}

$$M_{\mathrm{PDL}}(\varepsilon) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\varepsilon} \end{pmatrix}$$

where ε corresponds to the minimum transmission $T_{\min}$ ($T_{\max}$ is assumed to be 1), with $\mathrm{PDL}(dB) = \gamma_{\mathrm{loop}} = 10\log(1/\varepsilon)$. The total transfer matrix of the loop M_{loop} can hence be expressed as

$$M_{\mathrm{loop}} = M_{\mathrm{PDL}} \cdot M_{\mathrm{PC}}.$$

We want to study the probability density distribution of the accumulated PDL as a function of the number of circulations inside the loop. This implies we do not limit ourselves to the case of just one loop, but we consider an arbitrary number of circulations N. Because of the periodicity of the recirculating loop, the accumulated total transfer matrix T_{loop} is equal to

$$T_{\mathrm{loop}} = M_{\mathrm{loop}}^N = \underbrace{[M_{\mathrm{PDL}} \cdot M_{\mathrm{PC}}] \cdot \ldots \cdot [M_{\mathrm{PDL}} \cdot M_{\mathrm{PC}}]}_{N}.$$

In order to calculate the probability distribution for the different number of recirculations, we have to calculate the accumulated

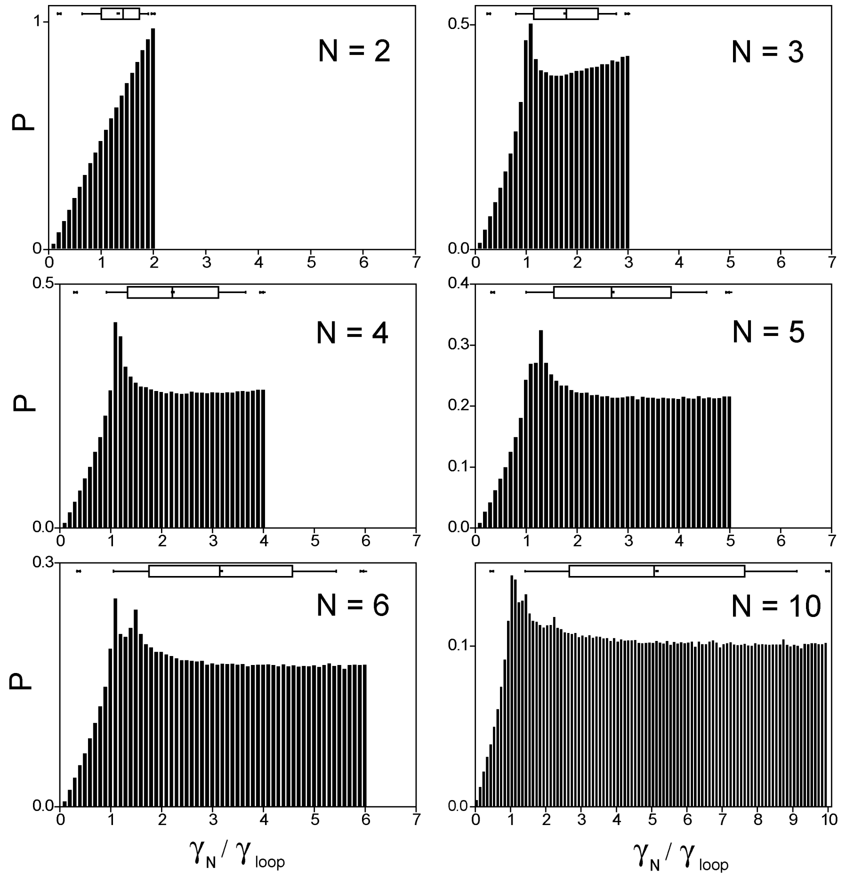

Fig. 1. Numerical simulations: probability density distribution for the accumulated PDL γ_N, normalized to the PDL loop value $\gamma_{\rm loop}$, for different number of recirculating loops (2–10). The total number of different loop configurations is equal to 10^6. A box chart is shown on top of each inset. The vertical lines in the box denote the 25th, 50th, and 75th percentage values. The error bars denote the 5th and 95th percentage values. The square symbol in the box denotes the average of the data.

PDL value corresponding to a large number of possible PC configuration that uniformly scramble the Poincarè sphere. This can be obtained, providing the angles θ and ϕ have the following probability distribution functions [20]

$$p(\theta) = \sin(2\theta) : \theta \in \left[0, \frac{\pi}{2}\right]$$
$$p(\phi) = \frac{1}{(2\pi)} : \phi \in [0, 2\pi].$$

Concerning the calculation of the PDL, in accordance with Heffner [21] the accumulated PDL value is equal to $\gamma = 10\log(\lambda_2/\lambda_1)$, where λ_i correspond to the eigenvalues of $T_{\rm loop}^{\dagger} T_{\rm loop}$. By writing $T_{\rm loop}^{\dagger} T_{\rm loop}$ in the form $pI + \boldsymbol{p} \cdot \boldsymbol{\sigma}$ one may show that $\lambda_{1,2} = p \pm |\boldsymbol{p}|$, as was used above.

The statistical nature of the accumulated PDL is thus determined examining 10^6 ensemble (i.e., 10^6 different PC configurations) and for each ensemble calculating the corresponding PDL values. In Fig. 1 is shown the probability density distribution for the accumulated PDL γ_N, normalized to the PDL loop value $\gamma_{\rm loop}$, for a different number of recirculating loops (2–10). When $\mathrm{N} = 2$ the probability distribution of γ_N is a linear function, in accordance with what found in [2] for the case of a concatenation of two PDL elements, and in [19] for two PMD elements. For larger N, the distributions tends to get uniform as evidenced by the box chart shown on the top of each distribution. This is in good agreement with the above theoretical discussion.

With regards to the simulations, two things are worth to be mentioned. First, as stated at the beginning, in our model we did not consider any PMD element in the loop. In fact simulations (not presented here) show that for small values of PMD, the PDL distribution results not to be affected by it (at least for the number of recirculations here considered). Second, when high values of PDL (>3 dB) are considered, the probability distribution will start to deviate from the one obtained for low PDL values. This is also in consistence with the fact that the vector-sum-concatenation rule is no longer valid in such a situation. Here, however, we are on the safe side, as we consider values of PDL in our experiment, always less than 3 dB.

III. Experimental

A. Setup

The setup of the experiment is shown in Fig. 2. Three acusto-optical switches (SW) are present in order to control the loop. A narrowband DFB laser (1550 nm) is used as a light source, and three different linear SOP are selected by rotating a half waveplate (HW) and a quarter waveplate (QW). The lightwave is then amplified through an Erbium doped fiber amplifier (EDFA) before passing through switch SW1 that controls the filling of the loop.

The basic principle of the optical loop is the following [12]. With the transmitter switch (SW1) closed and the loop switch

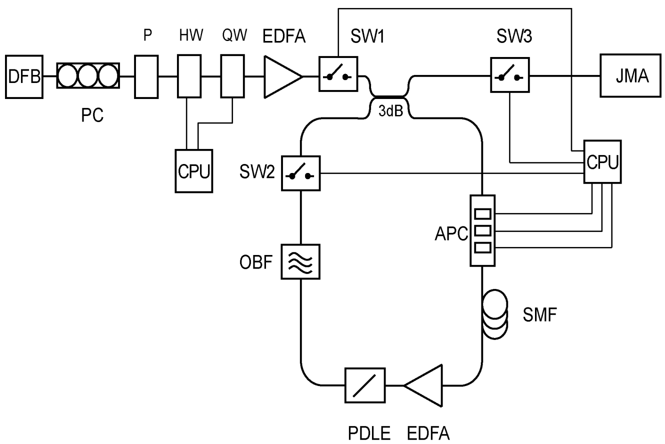

Fig. 2. Experimental setup for the recirculating loop. DFB distributed feedback laser, PC polarization controller, APC computer controlled PC, P polarizer, HW half waveplate, QW quarter waveplate, SW switch, JMA Jones matrix analyzer, PDLE polarization-dependent loss emulator.

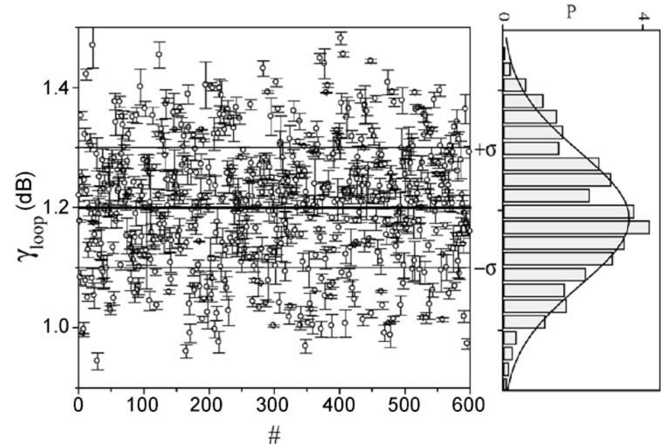

Fig. 3. PDL of the loop ($\gamma_{\rm loop}$) as a function of different polarization settings for the APC. Bold solid lines indicate the average PDL; thin lines indicate the average PDL plus/minus one standard deviation σ. On the right panel, the obtained probability density distribution for the PDL is shown. The bold line corresponds to the best fitting probability density Gauss function ($\chi^2 = 0.06$) with PDL= $(0.27 + / - 0.10)$ dB.

(SW2) open, half of the light is launched into the loop, through a 3-dB coupler. SW1 is kept closed for a time $\tau_{\rm fill}$ until the loop is filled. Then switch SW1 is opened and SW2 is closed and the light is allowed to circulate for a certain time $\tau_{\rm loop}$ that corresponds to N circulations of the loop. The state of the switches is then changed again and the experiment repeats. The extract switch (SW3) is opened with different timing with respect to SW1 and SW2 in order to extract the different pulses (i.e., for how many times a pulse is allowed to recirculate) from the recirculating loop.

As shown in the setup, different optical elements are present in the loop: a computer controlled polarization controller, a single mode fiber, an amplifier, a PDL emulator, a bandpass filter, and a coupler. The computer controlled polarization controller (APC) allows to mthe total Jones matrix of the loop. The 12-km SMF fiber acts as a delay line for the loop. The EDFA compensates for the power loss of each recirculation and its gain is set equal to the total loop loss. In order to avoid as much as possible any power step variation inside the loop, the frequency of the loop was set such that after N circulations, the loop is immediately filled with another pulse $\tau_{\rm fill}$. In this way the EDFA gain remains constant and the loop emulates correctly the response of a chain of EDFAs. The variable PDL emulator (PDLE) consists of an open beam launcher/collimator with a tilted glass plate inserted in between. Fresnel calculations relates the tilting angle of the plate with different values of PDL. The optical bandpass filter (OBF; $\rm FWHM = 1.3~nm$) centered on the DFB laser's wavelength, reduces the amplified spontaneous emission noise (ASE) of the EDFA, that will otherwise grow at each recirculation. Finally, the 3 dB coupler couples the light in and out the recirculating loop.

The PDL measurement principle (Jones matrix method) is based on the method of Heffner [21]. The method consist in measuring the polarization response of the device under test, to three different input SOP at a fixed wavelength. The PDL of the device under test (DUT), defined as in (3), is equal to $10\log(\lambda_2/\lambda_1)$, where λ_i correspond to the eigenvalues of $J^\dagger J$, with J the Jones matrix of the DUT. In our measurements we select three different linear states of polarization. After passing through a polarizer P, three linear SOP are obtained at 0°, 45° and 90° with each other, by rotating of 0°, and 45° the quarter waveplate QW and the half waveplate HW, alternatively. The output Stokes vectors are determined at the exit of SW3, and the Jones matrix of the recirculating loop and the corresponding PDL, are finally computed.

B. Results and Discussion

The probability density distribution of the accumulated PDL is obtained by repeating the following procedure. We first make measurements from two up to ten circulations in the loop, with the APC uniformly scrambled between the measurements. The PDL is measured 20 times for each polarization setting ($\sigma \leq 5\%$) rejecting the data outside one standard deviation from the average (typically 2–3). One thousand data (each one at a different polarization setting) are then measured for each number of circulations through the loop. From this data the probability density distributions are finally determined.

Before proceeding with the experiment, we initially tested the APC to see if it generates random uniformly distributed SOP on the Poincaré sphere. To do this, we generated a random series of voltages opportunely distributed, and we measured for 1000 different settings, the distributions of the Stokes parameters. The results evidenced that the Stokes parameters were all uniformly distributed, confirming the goodness of the scrambling procedure.

Another aspect we have to consider is that the polarization scrambler APC we use introduces a setting dependent PDL. This implies that the PDL of the loop ($\gamma_{\rm loop}$) is a function of the different polarization settings of the APC. The distribution of the PDL values for the APC is found to be Gaussian (when expressed in decibels), centered at 0.27 dB and with standard deviation equal to 0.1 dB.

Considering the nonnegligibility of this value, we set the PDL emulator at a value quite larger compared to the intrinsic PDL value of the loop. In Fig. 3 are plotted the PDL values of the loop $\gamma_{\rm loop}$ as a function of the different polarization controller

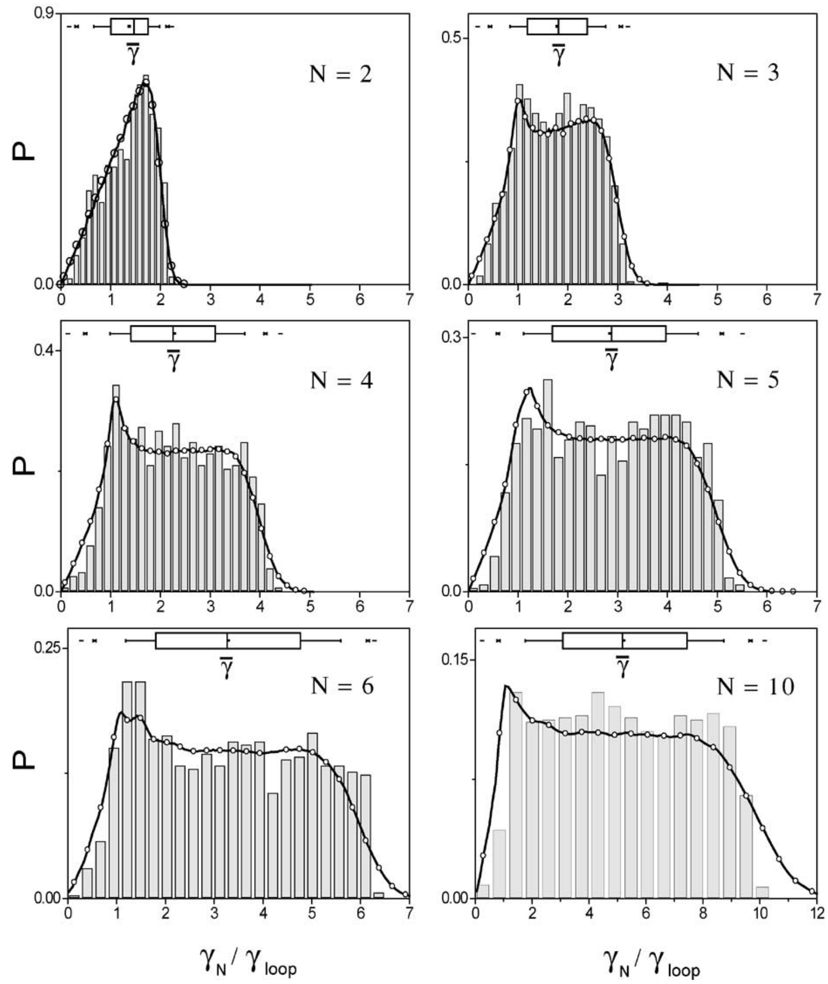

Fig. 4. Experimental: probability density distribution for the accumulated PDL γ_N, normalized to the PDL loop value $\gamma_{\rm loop}$, for different number of recirculating loops (2–10). The gray bars represent the experimental data with a total number of measurements equal to 1000. The bold line corresponds to the theoretical pdf. Experimental error bars are within the graphical resolution. Statistical error bars are not shown for clarity reasons. On the top of each inset is shown a box chart calculated on the experimental data.

settings. The bold solid line indicates the average PDL; the thin lines indicate the average PDL plus/minus one standard deviation σ. On the right panel is shown the obtained probability density distribution for the PDL. We assumed as a theoretical distribution a Gaussian probability distribution function and we then performed a goodness-of-fit Chi-squared test, a test commonly used to compare observed and theoretical (i.e., expected or assumed) frequencies. The low value for the reduced χ^2 (0.06) confirms that the data could be well described by an assumed Gaussian probability distribution function (bold line, in the right panel) centered at 1.20 dB, with a 0.10-dB standard deviation.

After having determined $\gamma_{\rm loop}$ (first circulation, $\rm N = 1$), we proceed as before switching out, using the extract switch SW3, the recirculating pulse in which we are interested. The probability density distributions for the accumulated PDL γ_N, normalized to the PDL loop value $\gamma_{\rm loop}$ are shown in Fig. 4 for different numbers of recirculating loops $(\rm N = 2 \ldots 6)$. The gray bars represent the experimental data with a total number of measurements equal to 1000. The bold line corresponds to the theoretical pdf obtained by convolution of the density function we found via numerical simulations and the Gaussian distribution function shown in Fig. 3. Both the experimental and the theoretical histograms, are sampled with the same bin width histogram parameter. As clearly seen from the figure, the agreement between the experimental and the theoretical results is excellent, confirming the validity of the model.

As mentioned in Section II-C the probability density distribution for the PDL is found to be equal to the one for the DGD. For comparison, in Fig. 5 are shown the experimental DGD probability distributions from another loop measurement [14]. Note the strong similarity between the distributions in Fig. 4 and Fig. 5, clearly showing that PMD and PDL follows the same distributions. One difference between the two theoretical pdfs is that the falloff of the PDL at high values is slower than for the DGD, and we attribute that to the Gaussian distribution of the PDL values. Simulations with constant PDL values show steep falloff, similar to the PMD case.

It is interesting to calculate the average value of the normalized accumulated PDL $\overline{\gamma} = \langle \gamma_N / \gamma_{\rm loop} \rangle$ as a function of different recirculating loops. The data are shown in Fig. 6. The filled triangles corresponds to the experimental data and are in good agreement with the theoretical ones (open circles). Hence, the accumulated average PDL increases linearly with the number of recirculations through the loop. This is in contrast to a randomized straight-line transmission link, for which the expectation value of the PDL grows as the square-root of

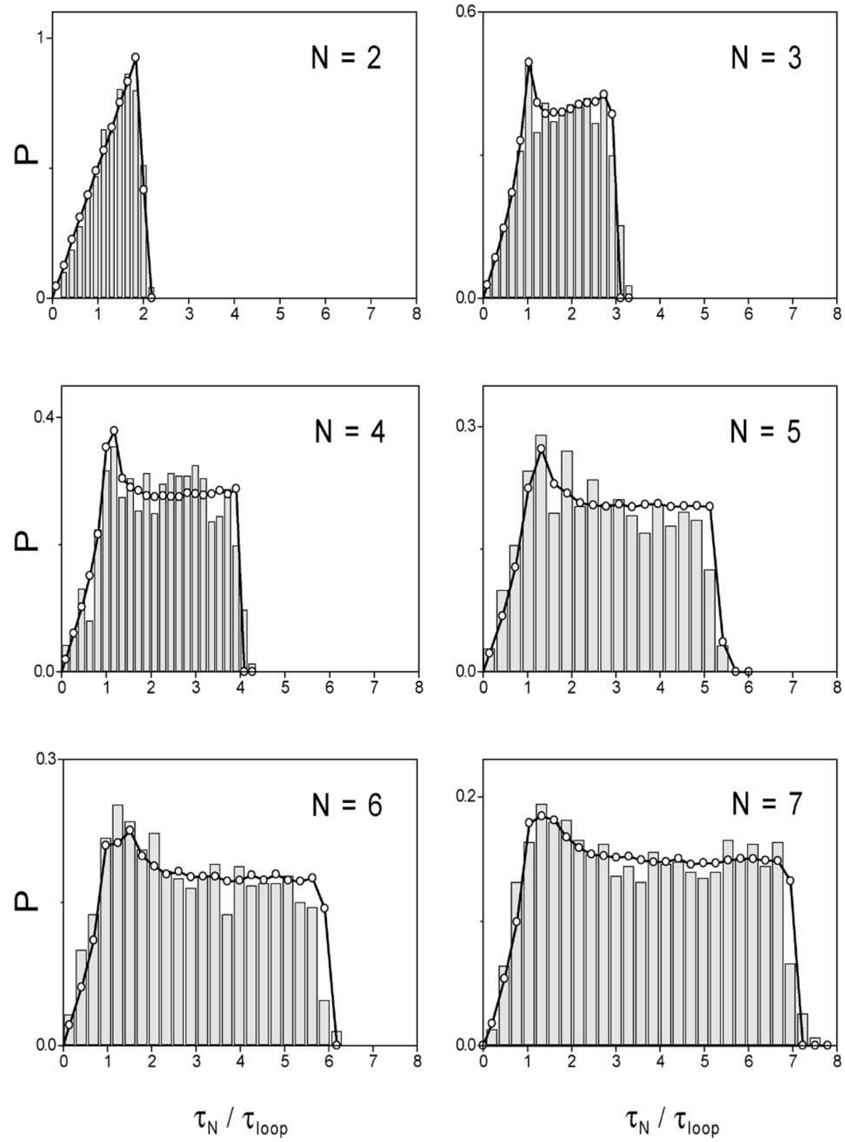

Fig. 5. Experimental: probability density distribution for the accumulated DGD τ_N, normalized to the loop value τ_{loop}, for different number of recirculating loops (2–7). The gray bars represent the experimental data with a total number of measurements equal to 2000. The bold line corresponds to the theoretical pdf. Experimental error bars are within the graphical resolution. Statistical error bars are not shown for clarity reasons. $\tau_{\text{loop}} = 1.6$ ps.

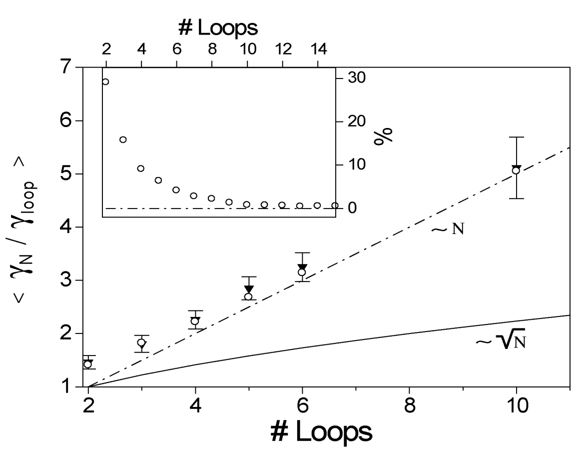

Fig. 6. Average normalized accumulated PDL $\overline{\gamma}(\langle\gamma_N/\gamma_{\text{loop}}\rangle)$ as a function of the number of recirculating loops. Open circles, calculated from theoretical pdf. Filled triangles, experimental data. Dot–dashed line, asymptote. Thin line, analytic behavior for a scrambled loop. In the inset is reported the deviation of the theoretically calculated total average PDL from the asymptote, as a function of the number of loops.

the link length (thin line) [3]. The dash-dot line in the figure indicates the asymptote toward which the data go, with an increasing in the number of loops. This is emphasized in the inset of Fig. 6, where the difference in percentage between the theoretical values and the linear ones, is plotted as a function of the number of loops.

Another parameter that is interesting to extract from the probability distributions is the root mean square normalized accumulated PDL $\sqrt{\overline{\gamma^2}} = \sqrt{\langle(\gamma_N/\gamma_{\text{loop}})^2\rangle}$. The values obtained for the different number of laps are shown in Fig. 7 (filled triangles). The bold line is the theoretical fit of the data using the analytic formula [see (15)]

$$\sqrt{\overline{\gamma^2}} = \gamma_{\text{loop}}\sqrt{\frac{N^2+2}{3}}.$$

From the fit ($\chi^2 = 0.0001$) we obtain a value for the loop PDL $\gamma_{\text{loop}} = (1.21 \pm 0.01)$ dB in complete accordance with

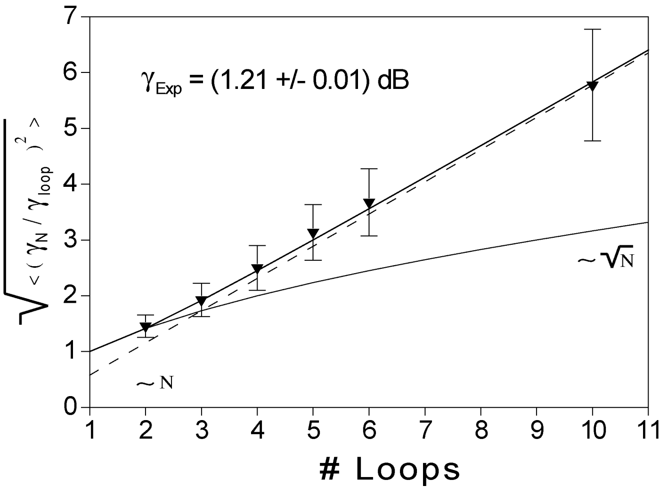

Fig. 7. Root mean square normalized accumulated PDL $\sqrt{\overline{\gamma^2}}(\sqrt{\langle(\gamma_N/\gamma_{\mathrm{loop}})^2\rangle})$ as a function of the number of recirculating loops. Filled triangles—experimental data. Bold line—theoretical fit of the data using the analytical formula. Dashed line—asymptotic behavior. Thin line—analytic behavior for straight-line randomized link.

the value (average of the Gaussian distribution) as measured independently for the first loop.

To note that in the loop the amount of measured PMD is found to be negligible, in agreement with the assumption made for the numerical simulations in Section II.

We note finally that for high values of N, the deviation of the experimental data from the theoretical ones, start to be quite important as evidenced by the experimental data for the tenth loop (last inset of Fig. 4). This can be ascribed to the presence of a residual, not filtered, ASE into the loop; contribution that becomes more important with the increasing number of recirculations.

IV. Conclusion

We have theoretically derived the statistical distribution of the PDL in a recirculating loop and we have found good agreement between the theory and the numerical simulations. An experimental investigation was also conducted and the measured probability density distribution is in very good accordance with the predicted one. We found that the probability density distribution for the accumulated PDL is a linearly increasing function for two circulations through the loop. For an increasing number of circulations, the PDL approaches instead a uniform distribution. We found also that for recirculating loops the mean accumulated PDL grows linearly with the number of circulations N, in contraposition to a straight-line optical system. For a recirculating loop, the statistics of PDL is found to be almost equal to the statistics of the DGD.

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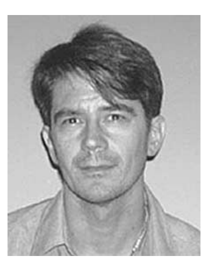

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