Distribution of differential group delay in recirculating loops

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Abstract: The statistical distribution of the differential group delay (DGD) in recirculating loops is investigated. The probability distribution of the DGD is obtained numerically and experimentally for two up to seven circulations through the loop.

Introduction

Recirculating loops (RL) are frequently used in research laboratories for emulating ultra long-haul transmission links [1]. To be able to reproduce the accurate Maxwellian DGD statistics of a randomly birefringent straight line link [2], the polarization has to be scrambled between each circulation inside the loop [3]. However, it is important to know the PMD statistics in RLs when no polarization scrambling is performed, as these loops are the most frequently used. Here, we investigate the probability distribution of the DGD in such loops. A polarization controller (PC) inside the loop is scrambled between each measurement of the accumulated DGD, but remains unchanged as the light circulates inside the loop. The distributions of the accumulated DGD after two, and up to seven circulations are derived from theory and by experiments.

Simulations and experiments

The RL consist of a PC and a PMD-element. The PMD-vector, $\mathbf{t}$, is fixed and its length and direction are given by the PMD-element. The PC corresponds to a rotation of the SOP an angle a around a rotation unit vector $\mathbf{r}$. The amount and direction of the rotation are determined by the PC setting. In order to accomplish correct polarization scrambling, one can show that the rotation angle a should be distributed as $p(a) = \frac{2}{\pi}\sin^2(\frac{a}{2})$; $a \in [0, \pi]$ and the angle b between $\mathbf{r}$ and $\mathbf{t}$ should be distributed as $p(b) = \frac{1}{2}\sin b$; $b \in [0, \pi]$ [4]. The PMD and the PC can be expressed by the Müller matrices M_{PMD} and M_{PC} and the total Müller matrix after N circulations through the loop becomes M_{RL}^N, where $M_{RL} = M_{PMD}M_{PC}$. The accumulated PMD-vector, $\mathbf{t}_N$, is then obtained and gives the accumulated DGD, $t_N = |\mathbf{t}_N|$ [5]. Since $\mathbf{t}_N$ is the sum of N equal-length PMD-vectors rotated around $\mathbf{r}$, the accumulated PMD-vector will spiral through Stokes space, as illustrated in Fig. 1a. Depending on the angle b, the spiral will be more or less extended with the extremes of $\mathbf{t}_N$ describing a circular movement perpendicular to $\mathbf{r}$, and $\mathbf{t}_N$ evolving along a straight line. By decomposing $\mathbf{t}$ in components parallel and perpendicular to $\mathbf{r}$, t_N can be expressed as

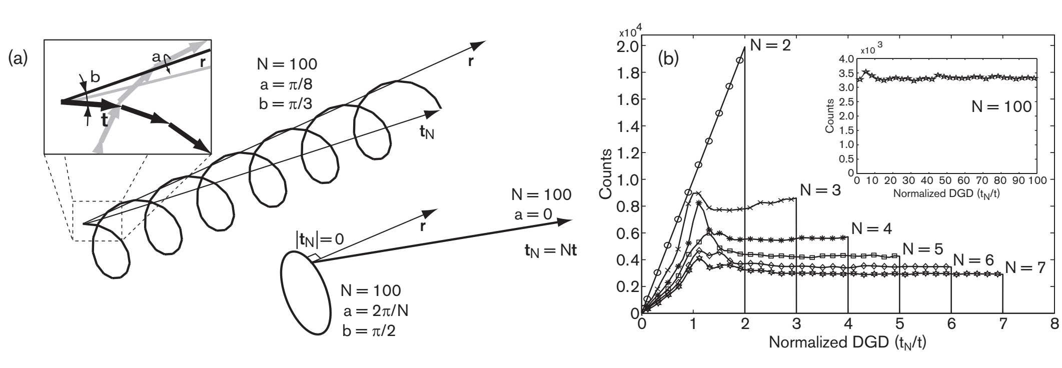

Fig. 1. (a) Typical evolution of the accumulated PMD-vector, $\mathbf{t}_N$, in Stokes space and the extremes resulting in $t_N = 0$ and $t_N = Nt$. (b) The distribution of the accumulated DGD for N = 2 - 7 and N = 100 (inserted). Number of runs 10^5.

$$t_N^2 = t^2 \left[N^2 \cos^2(b) + \sin^2(b) \frac{\sin^2(\frac{Na}{2})}{\sin^2(\frac{a}{2})} \right]. \quad (1)$$

By using Eq. 1 and the distributions of a and b, the distributions of t_N for different N were obtained. The results are shown in Fig. 1b. For $N = 2$ the probability distribution of t_N is a linearly increasing function. As the number of circulations increases, the DGD approaches uniform distribution, $p(t_N) = 1/(Nt)$, and for large N the expectation value of the DGD, $\langle t_N \rangle$, grows linearly as $\langle t_N \rangle = Nt/2$.

The RL was realized experimentally and is shown in Fig. 2. A tunable laser was used as the light source and the transmitted light was analyzed by a polarimeter. The loop consisted of a PC, a polarization maintaining fiber (PMF) with a DGD of 1.6 ps, an erbium-doped fiber amplifier (EDFA), a 1.3 nm optical filter (FWHM), and a 12 km standard single-mode fiber (SMF) and was controlled by three acousto-optical switches (AOS). The measured average PDL of the loop was 0.2 dB and the fluctuation of t was 0.1 ps. Measurements for different N in the RL were performed with the PC uniformly scrambled between the measurements. The DGD was computed according to the Jones matrix eigenanalysis [6] and roughly two thousand measurements of the DGD were performed for the different values of N. The results are shown in Fig. 2 together with the theoretical curve. The agreement between the experimental and the theoretical results is excellent.

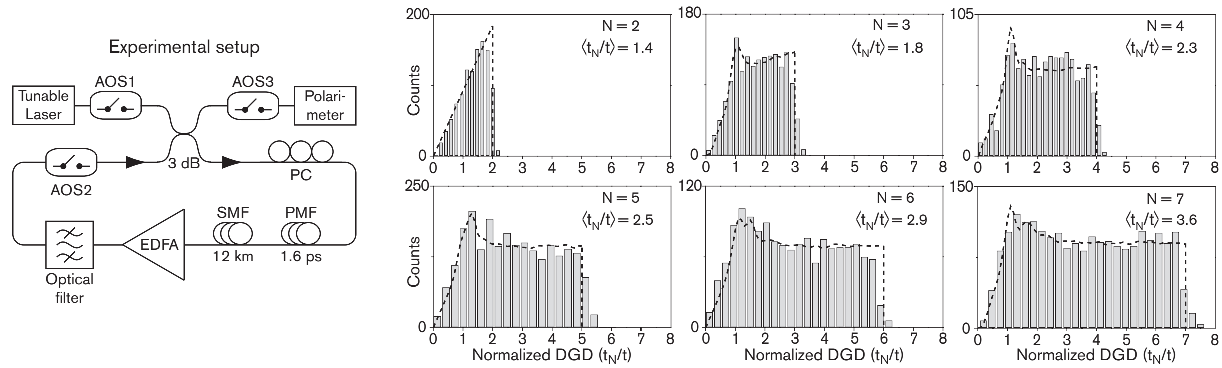

Fig. 2. Recirculation loop experimental setup and the measured histograms of the accumulated DGD for N = 2 - 7. Also, the normalized measured expectation value, $\langle t_N/t \rangle$, is presented. The dashed lines indicate the corresponding theoretical histograms. Number of measurements approximately 2000.

Conclusions

The statistical distribution of the PMD in a recirculating loop has been investigated and an analytical expression of the accumulated DGD has been derived. It was found that the accumulated PMD-vector evolves spirally through Stokes space and the accumulated DGD becomes uniformly distributed as the number of circulations increases. The expectation value of the DGD increases linearly with the number of circulations through the loop, in contrast to a straight line transmission link, where the expectation value of the DGD grows as the square-root of the link length.

References

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